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SENSITIVITY OF RADIOTELESCOPES AND CORRELATION RECEIVERS

E.J. Blum

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SENSITIVITY OF RADIOTELESCOPES AND CORRELATION RECEIVERS

Numerous models of telescopes have been suggested and employed for the observation of radioelectric radiation of the sky. It is essential to be able to compare them in order to select a type best adapted for a particular experiment. Up till now one has generally studied separately the characteristics of the receiver and those of the antenna. We are going to see that quite frequently it is not possible to draw a line of distinction. We shall demonstrate the existence of at least two classes of radiotelescopes which we shall call comparison and correlation apparatuses. In order for the comparison to be meaningful both the receiver and the antenna have to be considered. The study of correlation radiotelescopes has been relatively little developed and we shall try to show their qualities and their defects and insist specially on the problem of sensitivity.

Finally, we shall come forward with a simple technique for the realization of "correlation receivers."

II. PRELIMINARY REMARKS.

In all that follows we shall have to consider three distinct zones of the frequency spectrum:

1) The high frequency zone whose lower limit is around several mc. The antennas, the input stages and the medium frequency amplifiers of radiotelescopes function in this region of the frequency spectrum. The fluctuating voltages are random

stationary Laplace functions ("white" spectrum), and all the operations, realized in the radiotelescopes are, except for the detection, linear. The change of frequency operates a simple translation of the spectrum of fluctuating voltages without altering the energy properties. Therefore we shall not have to specify, generally speaking, the frequency in the vicinity of which the radiometer functions.

2) The zone of low frequencies of approximately 100 kc/to 10 cps;in this region the spectrum may be "white", but in general there appear effects of scintillation or fluctuation of gain with a growing spectrum towards the low frequencies.

3) The zone of very low frequencies, i.e. below several cps. The recording devices are usually sensitive to frequencies of this order.

It is known that the energetic properties of a fluctuating random /stationary Laplace voltage x (t) are entirely determined by its correlation function $\rho(\tau) = \overline{x(t) \cdot x(t-\tau)}$.

In the zone of high frequencies the fluctuating voltages pass through linear filters. These filters have a relatively broad band (of the order of mc). It is these filters that entirely determine the correlation function. In particular this function becomes negligible as soon as τ attains a value comparable to the time constant of the filters which is of the order of a microsecond.

One has therefore to realize that a fluctuating voltage after passage through such filters, being observed ~~at intervals of the~~ order of a millisecond, is absolutely incoherent with respect to itself or, to be more precise, $\rho(\theta) = 0$ if $\theta \gg 1/B$, where B is

the passband of the filters.

III. SENSITIVITY OF THE RECEIVERS (1).

The measuring apparatus on the output side of a radiotelescope indicates an average value and fluctuates around this value These fluctuations are characterized by their correlation function R (τ) and its Fourier transformed curve, the spectrum energy density A (f).

We shall normalize the gain of the systems so that a temperature variation of 1° of the antenna temperature may yield a variation equal to 1 with respect to the mean value as shown by the measuring device, and shall characterize the sensitivity of the radiotelescope by computing, under these conditions, A(0) or the spectrum energy density for a band of 1 cps and for frequencies f neighboring on 0.

We shall denote by T_A the temperature of the antennas and by T_R the apparent temperature of the receiver, conventionally fed back to the input resistance R. More often than not the temperature T_A will be much inferior to T_R: this is an interesting case in radioastronomy and we shall write $T = T_A + T_R$ with $T_A \ll T_R$.

We shall finally suppose a square passband of width B and the gain of the receiver g(f) to be equal to g for $f_1 \leqslant f \leqslant f_2$ ($f_1 - f_2 = B$) and 0 elsewhere.

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(1) It was the methods indicated by A. Leblanc-Lapierre (1949 and 1953) that spurred us on to study these questions. We refer the reader, once and for all, to these methods.

To the antenna temperature T_A there corresponds a fluctuating voltage $x_A(t)$, while $x_R(t)$ corresponds to T_R. These voltages have an average zero value $\overline{x_A}(t) = \overline{x_R}(t) = 0$. In order to know T_A it is necessary to measure averages of an order higher than 1 of $x_A(t)$, and consequently to utilize non-linear devices.

A. Standard detection.

On the output side of the receiver (Figure 1) before detection one has a voltage X(t) with

(1) $$\overline{X^2} = 4Rg^2 kTB,$$

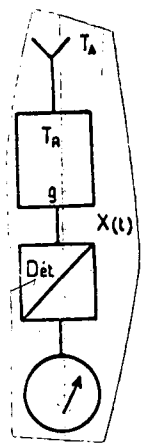

Figure 1.

its correlation function is

(2) $$\rho_x(\tau) = \frac{4Rg^2kT}{2\pi} \frac{\sin 2\pi f_2 - \sin 2\pi f_1 \tau}{\tau}$$

1) Quadratic detection: its characteristic is $y(t) = X^2(t)$. The average value of the detected voltage is

$$\bar{y} = \overline{X^2} = 4Rg^2kTB$$

If T_a varies by 1°:

$\Delta \bar{y} = 4Rg^2kB$ and, following the adopted normalization, $\Delta \bar{y} = 1$. Since the correlation function of the detected voltage is

$$\rho_y(\tau) = 2\rho_x^2(\tau),$$

the spectrum energy density on the output side can be written:

$$A(f) = 4\int_0^\infty 2\rho_x^2(\tau)\cos 2\pi f\tau \, d\tau.$$

if we consider the obtained spectrum by distributing the average power on the axis of positive frequencies only.

Or, at very low frequencies, i.e. for f approaching 0 and cos $2\pi/\tau \to 1$.

$$A(0) = 8\,\frac{16R^2 g^4 k^2 T^2}{4\pi^2}\int_0^\infty \left(\frac{\sin 2\pi f_2 \tau - \sin 2\pi f_1 \tau}{\tau}\right)^2 d\tau,$$

by replacing g by its value given by $\Delta\bar{y} = 1$

$$A(0) = \frac{2T^2}{B^2\pi^2}\int_0^\infty \left(\frac{\sin 2\pi f_2 \tau - \sin 2\pi f_1 \tau}{\tau}\right)^2 d\tau,$$

and

(3) $$A(0) = \frac{2T^2}{B}.$$

2) Linear detection with two alternations:

The characteristic is

$$y(t) = |X(t)$$

We then have

$$\bar{y} = 2\sqrt{\frac{\overline{X^2}}{2\pi}} \qquad \text{and} \qquad \Delta\bar{y} = 4g\sqrt{\frac{RkB}{2\pi}}\,\frac{T_A}{2\sqrt{T}}.$$

The correlation function of y(t) is

$$\rho_y(\tau) = \frac{\rho_X^2(\tau)}{\bar{y}\,\overline{X^2}}$$

and the spectrum energy density of the fluctuation of y(t) can be written

$$A(0) = \frac{4}{\pi\overline{X^2}}\int_0^\infty \rho_X^2(\tau)\,d\tau,$$

by replacing, as previously, g by its value obtained through writing $T_A = 1^o$ and $\overline{y} = 1$, we find:

$$A(0) = \frac{2T^2}{B} \quad \text{(second-order approximation)}$$

The linear and quadratic processes of detection have the same efficiency from the point of view of sensitivity at least under conditions we have put ourselves in. Moreover, since the characteristics of a linear detector are better defined and more stable than those of a quadratic detector, the latter is generally of no advantage. Finally, it is easy to realize that a linear detector rectifying only one alternation yields the same result. (1)

B. Correlation.

Attention to the possibility of such a type of detection has already been drawn by Tucker (1955) and Goldstein (1957) (2). But as we shall see later, several systems already in use actually function along this principle. (Rotatable or phase commutation interferometer, cross of Mills.) Ryle (1952) has shown that a phase commutation interferometer arrives at a product.

Let us consider 2 receivers with gains g and g_2 (Figure 2), their noise temperature T_R and T_{R_2} conventionally fed back to their resistances on the input side R_1 and R_2, their common

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(1) Analogous results have already been arrived at by Van der Zoel (1954) and Leroux (1955).

(2) See also Goldstein (1957), Strom (1957).

passband B, and 2 antennas connected each to one of the receivers.

The fluctuating voltages $x_{R_1}(t)$ and $x_{R_2}(t)$ corresponding to T_{R_1} and T_{R_2} are evidently not correlated, while the fluctuating voltages $x_{A_1}(t)$ and $x_{A_2}(t)$ corresponding to T_{A_1} and T_{A_2} may be correlated. We shall at first suppose that they are entirely correlated: $x_{A_1}(t) = x_{A_2}(t)$.

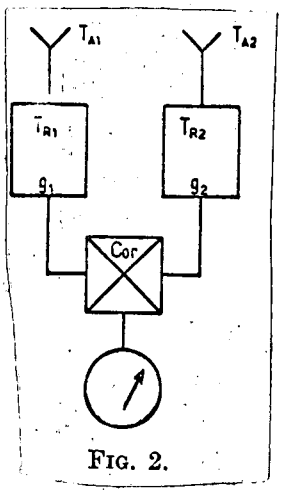

FIG. 2.

We shall come back later to the validity of this hypothesis. As in the case of standard detectors, we have on the output side of each of the receivers voltages $X_1(t)$ and $X_2(t)$, and their correlation functions $\rho_{x_1}(\tau)$ and $\rho_{x_2}(\tau)$, given by formulas (1) and (2).

The characteristic of the correlator (or multiplier) is:

$$y(t) = X_1(t) \cdot X_2(t)$$

a) Let us suppose at first $T_{A_1} = T_{A_2} = 0$

$X_1(t)$ and $X_2(t)$ are therefore independent and

$$\bar{y} = \overline{X_1(t) \cdot X_2(t)} = 0$$

The average voltage detected in the absence of the signal is zero. The correlation function of y(t) is

$$\rho_y(\tau) = X_1(t)\, X_1(t - \tau)\, X_2(t)\, X_2(t - \tau).$$

Since $X_1(t)$ and $X_2(t)$ are not correlated, one can separate the averages, and

$$\rho_y(\tau) = \rho_{x_1}(\tau)\, \rho_{x_2}(\tau).$$

b) Let us now introduce on the input side of both receivers 2 correlated voltages. In this case $\overline{y}$ is not zero anymore and we have:

$$\overline{y} = \overline{X_1(t).X_2(t)},$$

by taking in $X_1(t)$ and $X_2(t)$ only the correlated parts, or:

$$\overline{y} = 4Rg_1g_2kB\sqrt{T_{A_1}T_{A_2}}.$$

(in the supposition that B is identical for both receivers); since we have at our disposal 2 antennas we shall normalize by applying upon both antennas a power corresponding to an increase in temperature of 1°, or $.5^\circ$ apiece.

$$(4) \qquad \overline{y} = 1 = 4Rg_1g_2kB\frac{1}{2}.$$

On the other hand the spectrum energy density of fluctuations of y(t) can be written:

$$A(f) = 4\int_0^\infty \rho_{X_1}(\tau)\,\rho_{X_2}(\tau)\cos 2\pi f\tau\, d\tau,$$

and

$$A(0) = 4\,\frac{16R^2 g_1 g_2 k^2 T_{R_1} T_{R_2}}{4\pi^2}\int_0^\infty \left(\frac{\sin 2\pi f_2\tau - \sin 2\pi f_1\tau}{\tau}\right)^2 d\tau,$$

By replacing g_1g_2 by its value given in (4) we have:

$$A(0) = \frac{4T_{R_1}T_{R_2}}{B} \quad \text{et si} \quad T_{R_1} = T_{R_2} :$$

$$(5) \qquad \boxed{A(0) = \frac{4T_R^2}{B}}.$$

The spectrum density of fluctuations with this method of detection is therefore double the one yielded by a standard detection with the difference, however, that the fluctuations are not superimposed upon a continuous component stemming from noise in the receivers. We shall finally note that $\mathfrak{x}_{A_1}(t)$ and $\mathfrak{x}_{A_2}(t)$

may be correlated but of different signs, and $\bar{y}$ may take negative as well as positive values.

C. Double correlation and mixed systems.

To extend the preceding considerations let us now examine rapidly the sensitivity of a system in which we shall have 4 antennas, 4 identical receivers and a device allowing us to arrive at the product:

$$y(t) = X_1(t).X_2(t).X_3(t).X_4(t).$$

With the same notations and the same hypotheses as before, we see that $\varphi_y(\tau) = \varphi_{x_1}(\tau)\,\varphi_{x_2}(\tau)\,\varphi_{x_3}(\tau)\,\varphi_{x_4}(\tau)$ and that A(0) is proportional to

$$\frac{T_R^4}{B}$$

It is known that T_R is in general of the order of 10^3 °K. The sensitiveness of a double correlation receiver will therefore be 10^6 times inferior to that of a receiver of the 2 types studied previously; this entirely prohibits its use. In particular, if the composition by correlation of 2 antennas is possible (as we shall see later) in order to obtain a dummy antenna, the composition by correlation of 2 dummy antennas can not be envisaged since the sensitiveness of such a system would be very bad.

We can also find mixed or successive solutions: correlation followed by a standard detection, 2 detectors followed by a multiplier, 2 correlators or 2 standard detectors. We shall not study them in detail, but it is abundantly clear that with

our hypotheses the spectrum energy density of fluctuations will, on the output side, always be proportional to T_R^4/B. If, on the other hand, we have unequal passbands before and after the first operation or else antenna or receiver temperatures of the same order of magnitude, the preceding results are not valid anymore and the analysis becomes considerably more complicated: in particular it is not certain that after multiplication we shall still have a random stationary Laplace functions that would, as far as the detected part is concerned, correspond to the voltages coming from the antennas.

D. Remarks.

We know that in the correlation receiver $\overline{y} = 0$ when $T_{A_1} = T_{A_2} = 0$; on the other hand, in a standard detection apparatus $\overline{y}$ depends on R, g, T_R and B and is never zero. We shall therefore be obliged, at least once, in order to know $\overline{y}$, to give T_A an established value T_o, i.e. to compare $\overline{y}$ (with T_A unknown) to $\overline{y}_o$ (with $T_A = T_o$). This comparison may be made infrequently, and then we have the direct radiotelescope in its simplest form, or it can be made frequently: it is the case of systems, described by Decke (1946 and Ryle (1948).

IV. The problem of antennas.

We shall at first demonstrate with the aid of examples that a correct comparison of correlation receivers and comparison receivers is impossible as long as we disregard the antennas.

A. Interferometer.

Let us consider a point source passing in front of an

interferometer; we shall limit ourselves to the unidimentional problem.

a) Two antennas constitute a standard interferometer of length L. The fluctuating power at the point of junction of the cables stemming from the 2 antennas varies between 0 (voltages arriving in opposite phase to the 2 antennas) and $T_A = 1^\circ$. The measuring apparatus on the output side traces the fringes of interferences with an amplitude 1 and a normalized spectrum density $A(0) = 2T_R^2 / B$.

b) Two antennas, at the same distance L, are branched to 2 identical correlation receivers. When the voltages arriving upon the 2 antennas are in phase, the indication of the measuring apparatus will correspond to $T_A = 1^\circ$, then to $-T_A$ when the voltages are in opposite phase.

The amplitude of the fringes will be 2, and the normalized spectrum density becomes $A(0) = T_R^2 / B$ (for an amplitude 1). We can see that the sensitivity of the correlation system is better than that of the standard system; the comparison of the preceding equations (3) and (5) established on the strength of examining the characteristics of only the receiver led to an opposite conclusion.

B. Realization of a dummy antenna.

In a standard system the power response (1) of the antenna system is perforce equal to the square of the voltage response On the other hand, in the correlation system, the power response of the antennas is the product of the voltage response of the 2 special antennas, these antennas are different (that is

(1) Let be an antenna; in the presence of a "monochromatic" (i.e. such, that time does intervene in the expression of the

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(cont. of footnote)

electromagnetic field except by the factor exp $2\pi jft$) point source, situated in a direction marked by its directing cosines u, v, as related to an axis connected to the antenna, and producing near the antenna a field of known dimension, chosen for the definition of a reference level; the voltage within the boundaries of the antenna will be (close to the factor exp $2\pi jft$): $\gamma(u, v)$. We shall call $\gamma(u, v)$ the voltage response of the antenna. We shall also have to consider its square, which we shall call the power response.

to say not ~~mingled.~~) One will necessarily get a power response different from that given by a standard system. Let us illustrate this important property with an example:

Let us connect to one of the receivers a network of n equidistant antennas, the space between the antennas being L. (Figure 3). It is known that its voltage response can be written $g_i \frac{\sin n\Psi/2}{\sin \Psi/2}$, where $\Psi = \frac{2\pi L}{\lambda} \sin \theta$ and g_i is the individual response of an antenna. One will suppose that $g_i = 1$ and it will also be supposed that the angles θ between the direction of sight and the plane perpendicular to the network are very small.

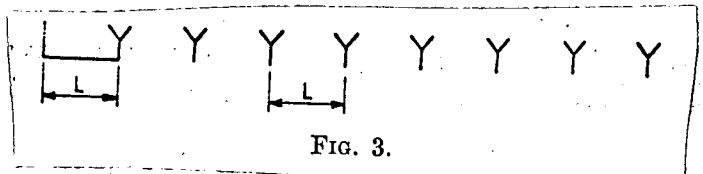

FIG. 3.

Let us connect to the other receiver a uniform antenna of length L, placed at the extremity of the network. Its response is of the form $\frac{\sin \Psi/2}{\Psi/2}$. Let us put our equidistant multiplier between the centers of the antennas. It will readily be seen that the product of the responses, i.e. the power response of the radiotelescope is of the form $\frac{\sin 2n\Psi/2}{\Psi/2}$ (at least in first approximation: the extreme antennas of the 2 networks cannot, in reality, be mingled.

This type of response gives the best average quadratic approximation for any distribution of celestial sources,

as shown by Arsac (1955). We could not attain such results with a standard telescope.

C. Correlation system with ~~mingled~~ antennas.

We have, in the previous paragraphs, supposed that the 2 antennas of the correlation systems were not ~~mingled~~. We shall now lift this restriction and examine the ensuing changes.

Let us take an antenna and divide the voltage appearing at its terminals in two equal parts which we will send into the 2 receivers of a correlation system. With the previously employed notation, the continuous voltage appearing at the terminal of the correlator can be written:

$$\overline{y} = \overline{(X_A(t) + X_{R_1}(t).(X_A(t) + X_{R_2}(t),}$$

and if $X_{R_1}(t)$ and $X_{R_2}(t)$ are independent, a summary analysis would permit us to write: (6) $\overline{y} = \overline{X_A^2}$; this is inexact. Actually the devices that permit us to direct a voltage to 2 paths can be:

1) "reciprocal" (Figure 4). A standard Y-junction for instance. In this case part of the noise $KX_{R_1}(t)$ passing at the terminals of receiver I reaches receiver 2 and vice versa. In reality we have:

$$\overline{y} = \overline{(X_A(t) + X_{R_1}(t) + KX_{R_2}(t))\ (X_A(t) + X_{R_2}(t) + KX_{R_1}(t))},$$

and formula (6) is false; in actual fact $\overline{y} = \overline{X_A^2} - \overline{X_o^2}$ if we call $X_o(t)$ the common part of $X_{R_1}(t)$ and $X_{R_2}(t)$.

2) "non reciprocal" (Figures 10 and 11), by which are meant unidirectional devices, magic T, ferrite lines for instance.

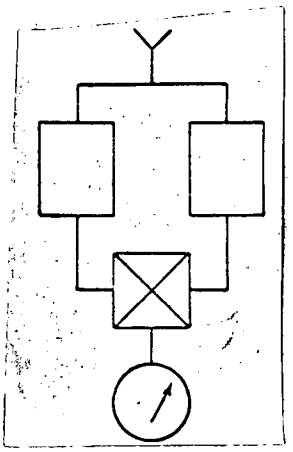

Figure 4.

These devices always possess an absorbing path which sets up a fluctuating voltage $X_o(t)$ common to both paths and we have to write $\overline{y} = X_A^2 - X_o^2$.

In these two cases we have in reality a comparison radiotelescope since we can only know the difference between the temperature of the antenna and a temperature T_o corresponding to the preceding voltage $X_o(t)$.

These examples demonstrate that the 2 methods are not really that much different if we consider the receivers only.

A correlation apparatus with one antenna is tantamount to a comparison radiotelescope. On the other hand a single conveniently permutated receiver with 2 systems of antennas will yield results comparable to correlation radiotelescope:Mills (1953), Ryle and Covington (1957) have thus realized synthetic antennas. But this always entails a loss of sensitivity as we shall see later.

V. The receiver - antenna "ensemble."

We have just seen that a separate examination of the characteristics of the antennas and the receiver often gives a rather inconvenient picture of the performances of a radiotelescope.

A method used at first in optics by Duffieux (1946) and Blanc - Lapierre and then in radioastronomy by Arsac and Le Roux (1955) is well suited for the simultaneous study of the properties of the antenna and of the receiver, and can be extended

to the case that interests us (1).

In order to introduce this method let us at first note the analogy between a linear electric filter and an antenna; in both cases we deal with linear systems which cause a response to correspond a signal depending on certain parameters (time t, frequency f or angles u, ν).

There are two methods of studying the response of a filter: an investigation of permanent conditions or also of transient conditions. The first method consists in considering a sinusoidal signal $\mathfrak{x}(f)$ of an amplitude A, of frequency f and its response y(f), linked to $\mathfrak{x}(f)$ by

$$y(f) = G(f).\mathfrak{x}(f). \tag{7}$$

G(f) being the gain of the filter.

In the second method to any signal X(t) there is a corresponding response Y(y). They are linked together by

$$Y(t) = \int_{-\infty}^{+\infty} R(t-t')\,X(t')\,dt', \tag{S}$$

R(t) being the response of the filter to Dirac's function, and we know that R(t) is the Fourier's transform of G (f).

In the same way let us consider the temperature of the sky (2) in a certain direction whose bearings are given in relation to axes linked with the celestial sphere: T (u, ν); if $u_o \nu_o$ give the bearing of the response antenna in power Γ (u - u_o, ν - ν_o) in

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(1) Suggestions of J. Arsac and E. Le Roux have been very useful in preparing this part of our survey.

(2) We suppose T (u, ν) to be invariant with time.

in relation to the same axis, the power appearing at the terminals of the antenna is:

$$U(u_0, v_0) = \int_u \int_v \Gamma(u - u_0, v - v_0)\, T(u, v)\, du\, dv. \tag{9}$$

In reality one can in general, with a sufficient approximation, assimilate the celestial sphere to its tangential plane in the direction of the antenna's axis, and the u, v are the rectangular coordinates in this plane.

Equations (8) and (9) are similar, and it is natural to define the "spatial gain" of the antenna, G(p,q) deducted from $\Gamma(u, v)$ by a transformation of Fourier.

$$G(p, q) = \int_u \int_v \Gamma(u, v)\, e^{2\pi j(pu+qv)}\, du\, dv,$$

It is also natural to write the response

$$B(p, q) = G(p, q) \cdot \beta(p, q),$$

by analogy with (7), β being a sinusoidal at spatial frequencies p and q.

We shall not go into greater detail into the conditons under which this transformation is possible (see Arsac 1955 and 1956); let us point out that it is simple for small angles u and v around the direction of sight of the antenna.

In the case of 2 antennas (and we have seen that it is not necessary, at least in radioastronomy, to envisage more than 2 antennas), there appears at the terminals of the first antenna a voltage:

$$c_1(u_0, v_0) = \iint \gamma_1(u - u_0, v - v_0)\, A(u, v)\, du\, dv.$$

A $(u,\vartheta)\,du\,d\vartheta$ being the electric field coming from the sky in the direction u,ϑ and $\gamma(u-u_0,\vartheta-\vartheta_0)$ being the response in voltage in the same direction (γ_1 may introduce a phase displacement).

For the second antenna:

$$e_2(u_0,v_0)=\iint\gamma_2(u'-u_0,v'-v_0)\,A(u',v')\,du'\,dv'.$$

Let us effect the product e_1e_2 and then let us take the mean value by integrating along the time constant of the receiver:

$$\overline{U(u_0,v_0)}=\overline{e_1e_2}=\iiiint\gamma_1(u-u_0,v-v_0)\,\gamma_2(u'-u_0,v'-v_0)\,\overline{A(u,v)\,A(u',v')}\,du\,dv\,du'\,dv'.$$

As the signals coming from two different directions of space are not correlated, $\overline{U}$ is zero for all the values $u\neq u'$, $\vartheta\neq\vartheta'$. We can therefore write:

$$(10)\qquad \overline{U(u_0,v_0)}=\iint\gamma_1(u-u_0,v-v_0)\,\gamma_2(u-u_0,v-v_0)\,T(u,v)\,du\,dv,$$

in reality: $A^2(u,\vartheta)-T(u,\vartheta)$.

By defining $\Gamma(u,\vartheta)=\gamma_1(u,\vartheta)\,\gamma_2(u,\vartheta)$ as the power response of the dummy antenna, we find again equation (9), but (10) is valid whether the antennas be distinct, partly or entirely mingled; let us note that since T and u are real functions, Γ is certainly also a real function.

We shall also suppose that $G(p,q)=\int\Gamma(u,\vartheta)e^{2\pi j(pu-q\vartheta)}\,du\,d\vartheta$, to be the spatial gain of the dummy antenna.

Let us demonstrate now that in certain cases it is very easy to obtain G(p,q). In order to simplify, we shall only envisage the unidimentional problem, with 2 antennas in the same plane and we shall place ourselves within the hypotheses of optics for the diffraction "ad infinitum", with a monochromatic wave giving a field of amplitude A(u)du in direction u.

Let us consider points M and M' of abscissae x and x' respectively in the plane of opening of each antenna (Figure 5) and affected by a certain lighting $S(x)$, $S'(x')$; if we take as origin of the time the crossing for the wave at point O and measure x and x' in wavelengths, the voltage at M can be written:

$$de_M = dx \int_v S(x)\, A(u)\, e^{2\pi j x u}\, du,$$
$$de_{M'} = dx' \int_v S'(x')\, A(u)\, e^{2\pi j x' u}\, du.$$

The contribution of a couple of points to the total response can be written:

$$\overline{dU} = dx\, dx' \int S(x)\, S'(x')\, T(u)\, e^{2\pi j(x+x')u}\, du,$$

and, since the system is linear:

$$\overline{U} = \int_x \int_{x'} \int_u S(x)\, S'(x')\, T(u)\, e^{2\pi j(x+x')u}\, du\, dx\, dx'.$$

This equation is analogous to (9) and the spatial gain of the dummy antenna G(p) is Fourier's transform of the response

$$\Gamma(u) = \int_x \int_{x'} S(x)\, S'(x')\, e^{2\pi j(x+x')u}\, dx\, dx'.$$

Let us change the variable $x = x$, $x + x' = p$; we than have

$$\Gamma(u) = \int_x \int_p S(x)\, S'(p - x)\, e^{2\pi j u p}\, dx\, dp$$
$$= \int_p e^{2\pi j u p} \left[\int_x S(x)\, S'(p - x)\, dx \right] dp.$$

We can see immediately that $\Gamma(u)$ is a Fourier transform, so that G(p), which we are trying to find, can be written:

$$(11) \qquad C(p) = \int_x S(x)\, S'(p-x)\, dx.$$

In order to know G(p) it is sufficient, for each value of p, to take all the couples of points on the antennas, (these couples being shifted by p), to assign to each couple a coefficient depending on the lighting, and to add up the couples.

There appears now an important property of the correlation systems: if $p < L$ (L being the distance of the edge of the antennas measured in wavelengths), G(p) is zero since there is no couple of points for these values of p. A correlation radiotelescope with distinct antennas is insensitive to spatial frequencies varying from 0 to L; that is to say, it will not account for a continuous background or slow variations of same. This property is not rigorously exact in the general case (diffraction on the edge of the antennas), but has been verified in practice with two antennas sufficiently spaced out.

Les us now consider the case of an interferometer and compute the spatial gain by the method that has been indicated; the result is represented on Figure 6, L being the distance of the center of the antennas, l their length.

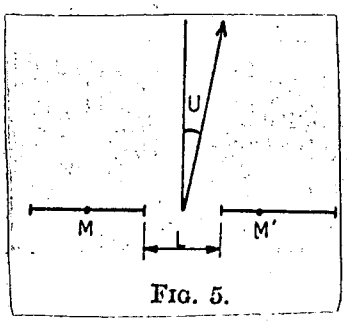


FIG. 5.

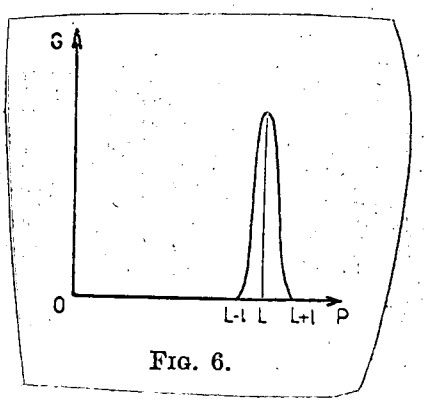


FIG. 6.

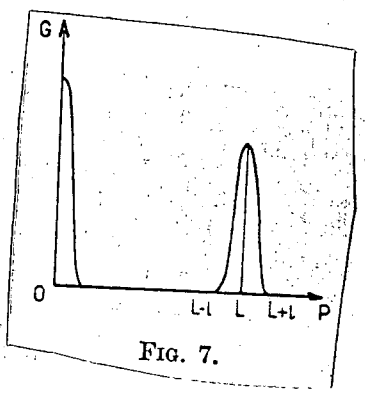


FIG. 7.

Insofar as a standard interferometer is concerned, formula (11) is still valid, but S = S' and one has to consider all the couples of points, including those on the antenna itself: the spatial passband is then split up in two: one part goes from 0 to ℓ, another, 2ℓ wide, centers around p = L (Figure 7).

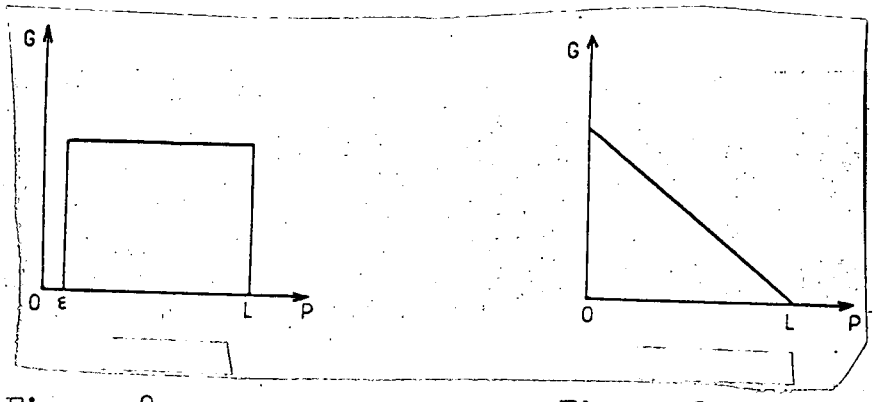


Figure 8.

Figure 9.

As in interferometry one tries in the first place to find frequency L, one can see that the difference in sensitivity which has been mentioned previously, is explained by the presence in the standard system of frequencies, neighboring on 0, which give unutilized information, that tends to degrade the sensitivity

Figures 8 and 9 respectively show the spatial gains of the dummy antenna of preceding page 11 (ε is the distance from the edge of the antennas nearest to the two networks, L is the total length) and of a standard uniform antenna of the same length. The dummy antenna gives much more information on high spatial frequencies, therefore a better separating power but does not let through frequencies neighboring on 0.

These considerations can be extended to the bidimentional problem and it is now possible to give a satisfactory answer about the two classes of radiotelescopes.

We shall call a correlation radiotelescope an apparatus that effects by any method, the product of voltages, appearing at the terminals of two distinct antennas, and a comparison radiotelescope an apparatus that raises to the square the voltage appearing at the terminals of one antenna. (We set aside in this work the case where the antennas of a correlation radiotelescope are partially mingled.

In the correlation systems we are not able to know the temperature of the background against which the measured area stands out; in the comparison systems it is necessary to compare the indication of the measuring apparatus to what it yields when the antenna is replaced by a source of known temperature.

We can also see how much the criterion of sensitivity used in paragraph III is arbitrary: the possibilities of comparison radiotelescopes and of correlation radiotelescopes are different and it is not, generally speaking, possible to compare their sensitivity with precision.

We shall now see, as we study different models of radiotelescopes, that it is often possible very simply to consider an amelioration of their characteristics. We shall, at the same time, investigate the influence of other factors that have so far not been mentioned yet: gain variation of the amplifiers and secondary lobes of the antenna.

VI. COMPARISON RADIOTELESCOPES.

A. Common characteristics:

1) Antennas. As has already been seen, the voltage response of the antenna is always raised to the square: this is favorable from the point of view of secondary lobes, but limits the types of possible circuits; a continuous background before the antenna gives a deviation of the measuring apparatus.

2) Receivers. Gain fluctuations: Dicke (1946) was the first to notice that phenomenon which has not so far been explained in a satisfactory manner. This phenomenon consists in a modulation of amplifying signals in the high frequency stages, having the same rhythm as the low frequency scintillation of the cathodes of the lamps. If convenient precautions are not taken, this effect combines with the "normal" fluctuations of the measuring apparatus. Steinberg (1949) has shown that the spectrum density of these fluctuations, as also that of scintillation, increases from several kc up to very low frequencies. It is, however, noticeable only from several hundreds of cps onward.

B. "Occasional" Comparison Radiotelescope.

Here we have an ordinary receiver, coupled by means of linear devices to a system of any antennas. The sensitivity is, as we have seen, characterized by $A(0) = 2T_R^2/B$. The relative variation of gain, which modifies by 1 the indication of the measuring apparatus, is given by

$$\frac{dg}{g} \quad \frac{1}{2T_R}$$

The system is accordingly very sensitive to gain variations

and an excellent stabilization of the circuits is necessary. In order to do this, a method developed at Cornell University, especially by Seeger (1950), yields very good results. It consists in the introduction of a strong counter-reaction of low frequency into the high frequency amplifying stages of the receiver. Experiments have shown that a comparison of the energy coming from the antenna to that coming from a standard source, made for several minutes every hour, is largely sufficient. The sensitivity is therefore characterized by the expression arrived at previously. But during the periods of gauging, observations are not possible. We shall therefore be led to say that the "utilizable" sensitiveness is not quite as good, the margin being a factor representing the time "wasted" during the gauging.

C. "Frequent" Comparison Radiotelescopes.

The systems, described by Dicke (1946) and Ryle (1948) with their numerous variants belong to this category. The input stage of the receiver is connected alternately to the antenna and to an independent source of noise. Technical considerations generally limit the rhythm of this permutation to several hundred cps. According to the remark made at the beginning of this survey, fluctuating tensions are entirely incoherent during 2 successive phases of the permutation.

During one half of the time one has (with a quadratic detection)

$$\overline{y}_1 = 4R_1 g^2 k(T_R + T_A)B,$$

and during the second half

$$\overline{y} = 4R_2 g^2 k(T_R + T_o)B,$$

T_o is the temperature of the reference source.

We suppose perfect permutation, that is to say without any dead time, and also immediate connection to the receiver of one and other of the sources, whichever is the most favorable case.

The correlation function of the fluctuations on the output side is always:

$$\rho_v(\tilde{\sigma}) = 2\rho_X^2(\tilde{\sigma}),$$

the difference is then

$$\overline{y}_1 - \overline{y_2} = 4Rg^2kB(T_A - T_o).$$

Let us suppose another perfect demodulation and, moreover, $R_1 = R_2$. For $T_A - T_o = 1^o$ we rediscover the same energy spectrum density as for receivers. Yet, since the measuring can only be performed during half the time, we have to write:

$$\overline{y}_1 - \overline{y}_2 = \frac{1}{2} 4Rg^2kB(T_A - T_o),$$

and the sensitivity is characterized by

$$(12) \qquad A(0) = \frac{8T_R^2}{B}$$

One would obviously find the same result with linear detection. The indication of the measuring apparatus varies by 1 for a gain change.

$$(13) \qquad \frac{dg}{g} \quad \frac{1}{T_A - T_o},$$

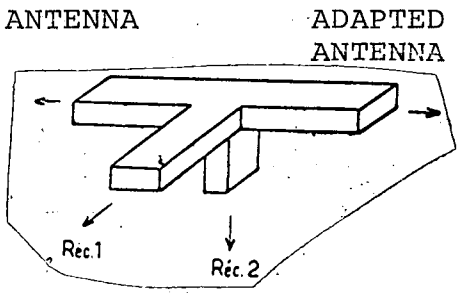
if one considers the gain variations as slower than the permutation frequency.

On the other hand the components of the gain variations, that are larger than the permutation frequency, introduce supplementary fluctuations upon the measuring apparatus whose amplitude may be equal to 1 for $\frac{dg}{g} = \frac{1}{\pi R}$. One may accordingly expect to obtain sensitivities that will be a little inferior to those given by formula (12).

And yet, expression (13) indicates that it will always be advantageous to make $T_A \sim T_\nu$: this is a permanent feature of the Ryle receiver and approximately the case of the receivers described by Mayer (1958) and Ewen (1958). Let it be pointed out that, in order to improve the sensitivity, one could resort to equalizing the durations of the connection of 2 sources: but one would face serious technical difficulties.

D. "Continuous" Comparison Radiotelescope.

We suggest this designation for what has been called previously " a correlation system with ~~mingled~~ antennas. Let us go back to the problem of connection of the antenna. We shall consider 2 methods:



1) one hybrid circuit of the magic T type separates the receivers (Figure 10). Figure 10.

Every receiver receives only one quarter of the power coming from the antenna and one quarter of the power coming from the adapted charge.

Under these conditions the average detected voltage is:

$$\bar{y} = 4Rg_1g_2kB\,\frac{T_A - T_o}{4},$$

and it can readily be found that for $T_A = 1^o$

$$A(0) = \frac{16T_R^2}{B} \quad \text{and} \quad \frac{dg_1}{g_1} + \frac{dg_2}{g_2} = \frac{1}{T_A - T_o}$$

with the adopted normalizations;

2) Two unidirectional circuits separate the receivers (Figure 11). A wave leaving point B towards a receiver encounters a characteristic impedance Z_o on every path. The circuit between the antenna and point B has accordingly to have a characteristic impedance $Z_o/2$ so that a maximum of power may reach the receivers. But the wave coming from the attenuator on the receiver side of the unidirectional lines propagates freely towards B over an impedance Z_o. It undergoes accordingly a reflect at B: one part goes towards the antenna and is radiated, the other 2 parts go freely towards the receiver. One has therefore at the input side of the 2 receivers a correlated component coming from the annenuators, and we shall assign to it a temperature T_o which we shall not attempt to specify. The result is therefore the same as that of the preceding paragraph except that the power received by each receiver is half the power available at the antenna terminals. This gives us:

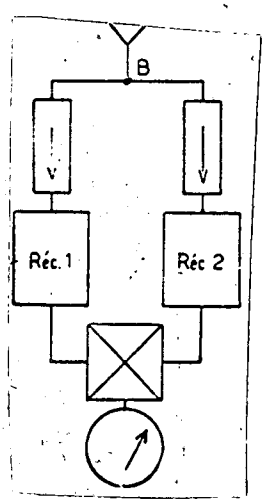


Figure 11.

$$\bar{y} = 4Rg_1g_2kB\,\frac{T_A - T_o}{2}$$

and the sensitivity is characterized by

$$A(0) = \frac{4T_R^2}{B}$$

The sensitivity to gain variations is of the form:

$$\frac{dg_1}{g_1}\,\frac{dg_2}{g_2} = \frac{1}{T_A - T_o}$$

irrespective of the frequency of these variations.

The latter system combines therefore the advantages of permutation apparatuses and a sensitivity which, although not as good as that of a direct receiver, is better than for permutation.

The influence of the gain variations is always weak: for considerable differences between T_A and T_o it is even less than in a permutation apparatus, since g_1 and g_2 vary generally very independently. On the other hand the introduction of a known temperature T_o, at least with the circuit we envisage, is a more delicate proposition.

VII. Correlation Radiotelescopes.

A. Common characteristics.

1) Antennas: we now have two systems of unmingled antennas.

We have seen that the response in power of the dummy antenna is

$$\Gamma(u, v) = \gamma_1(u, v)\,\gamma_2(u, v),$$

For a good utilization of the surface of the antennas it is advantageous to have $\gamma(U_o \nu_o) = \gamma_2(U_o \nu_o)$ in the direction of sight, or, in other words, to have equal surfaces for the 2 antennas. Only imperative reasons may lead to infraction of this rule for instance, when one of the antennas has to be mobile, as in the case of the dummy antenna of Ryle and Blythe (1957).

On the other hand, in the case of a dummy antenna with a single lobe the secondary lobes will be high. For instance in Mills's cross (1953) $\gamma_1 = \gamma_1(u), \gamma_2 = \gamma_2(\nu)$ and $\Gamma = \gamma_1(u)\gamma_2(\nu)$, that is to say that whatever would be given in voltage by each of the antenna will be yielded in power by the secondary lobes in each of the directions u and ν.

Finally, the condition of non-confusion of systems imposes a weak coupling between the two antennas: Mills was compelled to suppress, to this end, several dipoles at the center of his cross.

2) Receivers: the considerations developed with respect to gain fluctuations in "continuous and frequent comparison" systems can obviously be applied here in the case of "instantaneous" and non-instantaneous correlation, respectively.

B. Non-instantaneous correlation systems.

The phase permutation method, first used by Ryle, is tantamount to a correlation system.

Indeed, during half the time one detects the sum of fluctuating voltages coming from the 2 antennas and also that coming from the receiver; during the other half one detects the difference $X_{A_1}(t) - X_{A_2}(t)$ plus $X_R(t)$.

$$\overline{y_1} = \overline{\left(\frac{1}{\sqrt{2}} X_{A_1}(t) + \frac{1}{\sqrt{2}} X_{A_2}(t) + X_R(t)\right)^2}$$

$$\overline{y_2} = \overline{\left(\frac{1}{\sqrt{2}} X_{A_1}(t) - \frac{1}{\sqrt{2}} X_{A_2}(t) + X_R(t)\right)^2}.$$

The factor $1/\sqrt{2}$ corresponds in reality only to the junction of the 2 systems of antennas before the receiver; one can only receive the sum of the powers furnished by each antenna. Then one sets up the difference

$$\overline{y} = \overline{y_1} - \overline{y_2} = \overline{2X_{A_1}(t)\ X_{A_2}(t)}$$

Since the measurement can only be made during half the time, one has in reality:

$$\overline{y} = \overline{X_{A_1}(t).X_{A_2}(t)}$$

But the correlation function of the fluctuations after detection is always $\rho_y(\tau) = 2\rho_x^2(\tau)$, since $X_R(t)$ corresponds to a single receiver only.

C. Instantaneous correlation system:

By this is meant the 2-receiver system followed by a multiplier.

As has been seen before, for a detected voltage

$$\overline{y} = \overline{X_{A_1}(t).\ X_{A_2}(t)},$$

the correlation function of the fluctuations of $y(t)$ is given by $\rho_y(\tau) = \rho_x^2(\tau)$

Since the energy spectrum density is proportional to the correlation function of the fluctuations on the output side, it can be seen that with identical antennas an instantaneous correlation system will always be twice as sensitive as a non-instantaneous system.

VIII. EXAMPLES OF SENSITIVITY COMPARISON.

According to the remark made on page 22 we can talk with precision about sensitiveness only when we specify the type of antenna that is being utilized. In individual cases we can, on the other hand, gather the results obtained in order to give sensitivity values that the same antenna may attain while being operated by different methods.

1) Interferometer with 2 identical antennas and point source.

For the same amplitude of an interference fringe the energy spectrum densities are proportional to

T^2/B for a frequent comparison radiotelescope,

$2T^2/B$ for a non-instantaneous correlation radiotelescope or for an occasional comparison radiotelescope,

$8T^2/B$ for a frequent comparison radiotelescope.

2) Detection of point source with a system of single lobe antennas of total fixed surface; this is a very artificial example, but it permits appreciation of the utilization of the antenna surface in the diverse systems. For the same indication of the measuring apparatus the energy spectrum densities

are proportional to:

$8T^2/B$ for a frequent comparison radiotelescope with a single antenna, or a non-instantaneous correlation radiotelescope with 2 cross-shaped antennas of total surface S (Mills's Cross type),

$4T^2/B$ for a continuous comparison radiotelescope with a single antenna, or an instantaneous correlation radiotelescope with 2 cross-shaped antennas,

$2T^2/B$ for an occasional comparison radiotelescope with a single antenna.

3) Measurement of the brilliancy distribution of a source with an interferometer.

As is generally known, the amplitude of the fringes given by an interferometer measures a component of Fourier's transform of the brilliancy distribution of a source; the ensemble of the components is furnished by a series of differently spaced-out observations.

During these observations the "frequency" of the fringes is not utilized and can accordingly take on any value. If it is high, it compels the time constant of the measuring apparatus to be diminished which leads to a loss of sensitivity. Finally, when this frequency is of the order of magnitude of the passband of the receiver, it is necessary to perform another detection in order to be able to measure the amplitude of the fringes (Figure 12).

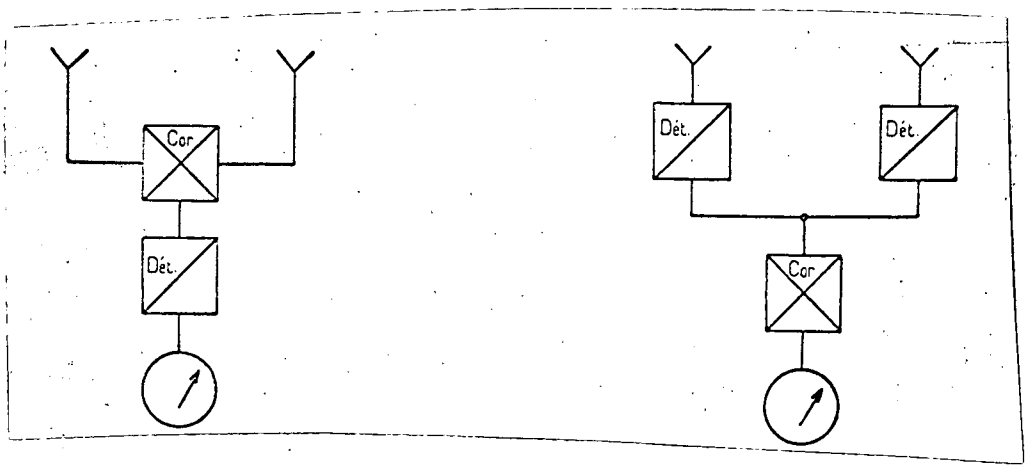


Figure 12.

Figure 13.

We rediscover a mixed correlation - detection system and the measurements are only possible with a power reaching the antennas of an order of the power of the noise of the receiver.

The radio interferometer conceived by Handbury-Brown is in analogous conditions (passband of approximately 2 kc, fringe frequency determined by the variations of local independent oscillators), but the order of the operations is reversed: detection precedes correlation; precisely this makes it possible to have independent oscillators (Figure 13).

If, however, technical reasons do not preclude the transmission of the signal with a relatively wide band over great distances, it will be preferable to put a correlator without preliminary detection at the center of the interferometer, then to filter the output side of the correlator before measuring with a band that is just wide enough to pass the maximum frequency of the fringes(this frequency being determined by the length of the base and augmented by the

propagation irregularities) and then to record the fringes themselves.

IX. REALIZATION OF A MULTIPLIER.

The two preceding comparisons have shown us anew the advantage of the method of correlation.

It will always be advantageous to replace:

1) a frequent comparison radiotelescope by a continuous comparison radiotelescope;

2) a non-instantaneous correlation radiotelescope by an instantaneous correlation radiotelescope.

We have not yet envisaged the technical problem that is posed by the realization of correlation apparatuses, and it is quite obvious that the thus realized advantages of sensitivity must not be paid for by a greater complexity.

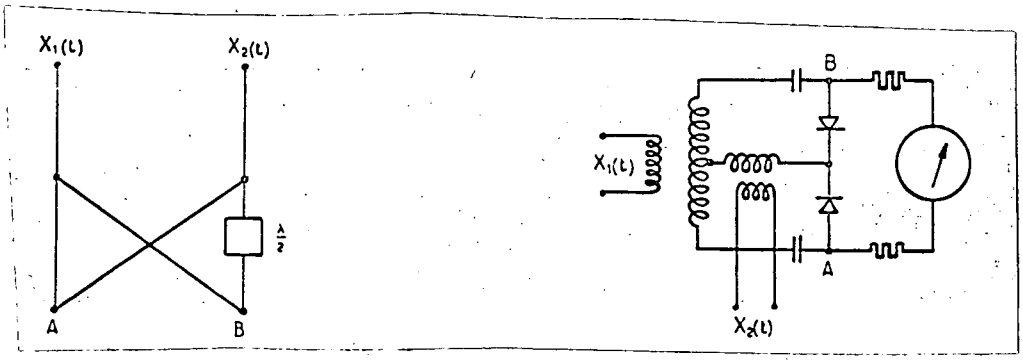


Figure 14.

Figure 15.

We shall see that this greater complexity is not the case and we are going to suggest a simple technique.

Let $X_1(t)$ and $X_2(t)$ be the two fluctuating voltages on band B. We are searching for this product.

Let us separate $X_1(t)$ into 2 equal parts: $\frac{1}{2} X_1(t)$, and $X_2(t)$ into 2 equal but opposed parts: $\frac{1}{2} X_2(t)$ and $-\frac{1}{2} X_2(t)$. Let us add at A: $\frac{1}{2} X_1(t)$ and $\frac{1}{2} X_2(t)$, and at B, $\frac{1}{2} X_1(t)$ and $-\frac{1}{2} X_2(t)$ (Figure 14).

Let us finally take the precaution of realizing the separation in such a way as to render the interaction between the 4 paths negligible: we can either put 4 equal attenuators in the 4 paths or perform the separation with the aid of an active circuit (double lamp) or a hybrid circuit (Figure 15).

Let us finally put at A and B two detectors which we shall at first suppose to be square-law ones with a characteristic $Y(t) = X^2(t)$

We have

$$Y_1(t) = \frac{1}{4} (X_1(t) + X_2(t)^2 \qquad Y_2(t) = \frac{1}{4} X_1(t) - X_2(t)^2.$$

Let us take the difference over the whole band B, which gives us:

$$Y_1(t) - Y_2(t) = X_1(t) X_2(t).$$

This result is well known.

Let us take now the linear detectors with 2 alternations, which leads to:

$$\overline{Y_1} = 2\sqrt{\frac{(X_1(t) + X_2(t))^2}{8\pi}} \qquad \overline{Y_2} = 2\sqrt{\frac{(X_1(t) - X_2(t))^2}{8\pi}}.$$

Let us then make the assumption of weak signals $X_{A_1}(t), X_{A_2}(t)$ before the background noises of the receivers $X_{R_1}(t)$, $X_{R_2}(t)$, or

$$X_1(t) = X_{R_1}(t) + X_{A_1}(t), \qquad X_2(t) = X_{R2}(t) + X_{A_2}(t)$$

and

$$\overline{x_{A_1}^2},\ \overline{x_{A_2}^2} \ll x_{R_1}^2,\ x_{R_2}^2,$$

One can then write:

$$\overline{Y_1} = \frac{1}{\sqrt{2\pi}}\sqrt{\overline{X_1^2}+\overline{X_2^2}}\sqrt{1+\frac{(\overline{X_{A_1}}+\overline{X_{A_2}})^2}{\overline{X_1^2}+\overline{X_2^2}}} = \frac{1}{\sqrt{2\pi}}\sqrt{\overline{X_1^2}+\overline{X_2}}\left(1+\frac{1}{2}\frac{(\overline{X_{A_1}}+\overline{X_{A_2}})^2}{\overline{X_1^2}+\overline{X_1^2}}\right).$$

and

$$\overline{Y_2} = \frac{1}{\sqrt{2\pi}}\sqrt{\overline{X_1^2}+\overline{X_2}}\left(1+\frac{1}{2}\frac{(\overline{X_{A_1}}-\overline{X_{A_2}})^2}{\overline{X_1^2}+\overline{X_2^2}}\right),$$

whence

$$\overline{Y_1}-\overline{Y_2} = \frac{1}{\sqrt{2\pi(\overline{X_1^2}+\overline{X_2^2})}}\,2\overline{X_{A_1}(t)\ X_{A_2}(t)},$$

which is the product that is being sought.

One would find an analogous formula for a rectifier with one alternation, and what has been seen on page 5 obviously applies here: the square-law and linear detectors in the wiring and with the indicated assumption perform the multiplication with the same efficiency from the viewpoint of sensitivity.

REMARK:

1) The realization of the multiplier which we have just described is simple; the only delicate point consists in constructing and maintaining two identical detector circuits. It is easy to make sure of this equality by connecting the input side of the 2 receivers to independent charges: the detected continuous voltage should be zero.

2) The phase rotation produced by the 2 receivers should be the same: this condition is not difficult to fulfill (Blum and al,. 1957).

3) The complexity of an apparatus with 2 receivers is certainly greater than that of a single receiver system. But it can be compared to the complexity of permutation instruments, where the construction of the commutator and of the phase detector is rather critical.

X. CONCLUSION.

A comparison radiotelescope and a correlation radiotelescope gain distinction only when an optimum of sensitivity has to be reached. On the other hand, the types of observations which they permit are never identical and one may view them as 2 distinct methods of measurement.

In the majority of cases, not only in radioastronomy, but in the whole field of electromagnetic waves, the methods of measurement derive from a comparison method: the systematic application of the correlation method appears interesting, especially in optics.

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